Abstracts of main talks

About a filtration of the group algebra attached to a reflection group

Cédric Bonnafé
Université de Montpellier

If W is a reflection group acting on a vector space V, then one can filter the group algebra (and so, any of its subalgebras) of W according to the codimension of fixed points. On the other hand, the d-Harish-Chandra theory of Broué-Malle-Michel, combined with the concept of Lusztig families of unipotent characters, allows to define morphisms between subalgebras of the center of different reflection groups. We conjecture these morphisms respect the above filtration and we give examples in which we know this conjecture holds. We also explains that this conjecture comes from an analogy between the combinatorics of unipotent representations and the geometry of Calogero-Moser spaces.

McKay's equality on p'-characters, the landscape after the proof, I

Marc Cabanes
Institut de Mathématiques de Jussieu

For any prime p, a finite group has as many irreducible complex characters of degree prime to p as the normalizers of its Sylow p-subgroups. This equality, conjectured by John McKay in 1971, was reduced in 2007 by Isaacs–Malle–Navarro to a conjecture on representations, linear and projective, of finite simple groups. In this first talk we comment about the context at the time of the reduction theorem, then on the consequences of the reframing by Späth first as a criterion expressed in terms of projective representations, then as a three part criterion for simple types with non-cyclic outer automorphism group. This leads us to a Lie-theoretic effort for which we introduce the main themes : the generalized Harish Chandra theory (Broué–Malle–Michel), the Jordan decomposition of characters (Lusztig), the descent equalities (Shintani), and other tools or updates devised for the conjecture itself. Some by-producs of independent interest like the determination of Irr(G) as an Out(G)-set for any simple group G or the extendibility of stable characters in many important normal inclusions, have benefited the proof of other glocal-local conjectures (Brauer height zero, Alperin–McKay for the prime 2) and is used in the ongoing effort towards other statements (Alperin weight conjecture, Dade conjecture) also reduced to questions about simple groups.

Braid group actions on Deligne–Lusztig varieties

Olivier Dudas
Aix-Marseille University

For a finite reductive group, two kinds of objects play a central role in representation theory. On the algebraic side one has irreducible characters; on the geometric side one has character sheaves. These two worlds are very close, and in the unipotent setting the passage from one to the other is governed by Lusztig's non-abelian Fourier transform.

Until now, this Fourier transform has essentially been computed through the classification: one classifies the relevant irreducible characters and character sheaves, and then determines explicitly the change-of-basis matrix between them. In ongoing joint work with C. Bonnafé, M. Broué, G. Malle, J. Michel and R. Rouquier, we develop a new approach to this transformation, which aims to explain it without using the classification.

The idea is to use Deligne–Lusztig varieties and the symmetries of their cohomology. More precisely, certain braid group operators act naturally on Deligne–Lusztig cohomology; in the simplest case, they recover the usual Hecke operators acting on the permutation representation of a finite flag variety. We show how to use the traces of these braid operators to recover the non-abelian Fourier matrix.

In this talk I will explain the geometric origin of these operators, some of their symmetries, and how recent categorical results on finite reductive groups, especially work of Eteve, provide the natural framework for this construction.

Jordan decomposition for finite groups of Lie type

Arnaud Eteve
MPIM Bonn

Let $\mathbf{G}$ be a reductive group over a finite field $\mathbb{F}_q$ and $G = \mathbf{G}(\mathbb{F}_q)$ be its group of rational points. One of the main results concerning the structure of the category $\mathrm{Rep}_k(G)$, where k is an algebraically closed field of characteristic 0 or $\ell \neq \mathrm{char}(\mathbb{F}_q)$, is Jordan decomposition which aim is to reduce most (if not all) structure questions to that of (ℓ-)unipotent representations. The goal of this talk is to present a complete version of Jordan decomposition, which is valid for modular representations, extends the results of Broué-Michel and Bonnafé-Dat-Rouquier and, in particular, resolves the problem of quasi-isolated elements. The methods we use however do not only rely on Deligne-Lusztig theory but rather use the modern language of categorical traces.

Generalised e-Harish-Chandra theory and Alperin weight conjecture

Zhicheng Feng
Southern University of Science and Technology

Harish-Chandra theory is a significant tool in Lie theory, such as in the representation theory of Lie groups, Lie algebras and finite reductive groups. In this talk, we will discuss a generalisation of e-cuspidality in the generalised e-Harish-Chandra theory (Broué–Malle–Michel) of finite reductive groups, and define the generic weights, which play an analogous role as the weights defined by Alperin in the investigation of the inductive Alperin weight condition for simple groups of Lie type at most good primes. Based on recent joint work with Gunter Malle and Jiping Zhang.

Special Chevalley bases for simple Lie algebras

Meinolf Geck
Universität Stuttgart

This talk will be quite elementary. Let $\mathfrak{g}$ be a finite-dimensional simple Lie algebra over the complex numbers, with root system Φ. We address some questions concerning the various possible choices of Chevalley bases for $\mathfrak{g}$, and the determination of the corresponding structure constants. In the simply laced case, Frenkel and Kac found a particularly simple construction of $\mathfrak{g}$, together with a Chevalley basis, in terms of a certain multiplicative 2-cocycle $\varepsilon\colon \mathbb{Z}\Phi \times \mathbb{Z}\Phi \to \{\pm 1\}$. We show that Lusztig's canonical basis of $\mathfrak{g}$ can also be obtained in this way, for a suitable choice of ε. We also address the problem of explicitly describing the structure constants when Φ is not simply laced.

Positivity of canonical bases

Xuhua He
The University of Hong Kong

Lusztig's theory of canonical bases reveals a remarkably rigid and positive algebraic structure on quantum groups and their modules. In symmetric types, it is known that the structure constants for multiplication in the negative part U^-, as well as for the action of Chevalley generators E_i and F_i on a single simple module, all belong to $\mathbb{N}[v, v^{-1}]$.
Lusztig conjectured that this strong positivity holds for the multiplication within the modified quantum group and the action on the tensor product of modules. In this talk, I will present recent joint work with Jiepeng Fang towards this conjecture.
A key innovation in our approach is the "thickening philosophy", an algebraic technique inspired by geometric ideas from total positivity, building on my earlier work with Huanchen Bao. This method embeds a suitable approximation of the tensor product into the negative part $\tilde{U}^-$ of a larger quantum group, constructed via a framed quiver. This allows us to inherit the desired positivity directly from the well-established positivity of the canonical basis of $\tilde{U}^-$. This approach demonstrates how the large Kac-Moody groups can provide a powerful framework for elucidating the structure and representations of quantum groups even for the finite and affine types.

Centralizers of semi-simple elements are semidirect products

Jean Michel
Université Paris Cité

This is joint work with François Digne.

Let G be a reductive group over an algebraically closed field k, and $s \in G$ a semisimple element. We show that there is a finite subgroup A of $C_G(s)$ such that $C_G(s) = C_G(s)^0 \rtimes A$.

In a second step, we study the same problem for finite reductive groups, that is when k is the algebraic closure of a finite field F_q, for the points of G defined over F_q.

From p-groups to defect groups to nilpotent blocks

Gabriel Navarro
Universitat de València

Let B be a p-block of a finite group with defect group D. A guiding principle behind some of the important problems in representation theory is that D relates to $\mathrm{Irr}(D)$ as D relates to $\mathrm{Irr}(B)$. We review some situations in which this principle holds and present both theorems and counterexamples. The celebrated Broué–Puig nilpotent blocks, however, always behave perfectly.

Beyond Brauer's Height Zero Conjecture

Noelia Rizo
Universitat de València

In 1963, Richard Brauer published a list of 43 problems that have shaped the development of the representation theory of finite groups ever since. Among them is the celebrated Brauer Height Zero Conjecture, which was positively proven very recently by Malle, Navarro, Schaeffer Fry, Tiep, and Ruhstorfer. Now that its validity is established, investigating extensions is essential to deepening our understanding.

In this talk, we discuss two such directions. The first is the Eaton–Moretó conjecture, which extends the Brauer Height Zero Conjecture to higher heights. Although still widely open, this problem has attracted considerable attention in recent years; we will focus on recent developments The second direction involves the action of Galois automorphisms on characters.

This talk is based on joint work with G. Malle, A. Moretó, M. Schaeffer Fry, and G. Souza.

Some elements of the Green ring related to Alperin's Weight Conjecture

Geoffrey Robinson
University of Aberdeen

We will discuss some virtual projective modules in the Green Ring of RG which are related to Alperin's Weight Conjecture (G a finite group, R the "usual" complete discrete valuation ring), and which give a module-theoretic perspective to the original numerical conjecture.

The Abelian Defect Conjecture at 40

Raphaël Rouquier
University of California, Los Angeles

I will discuss the role Broue's abelian defect group conjecture has played in the development of representation theory over the past forty years.

Galois automorphisms and blocks covering unipotent blocks

Lucas Ruhstorfer
Bergische Universität Wuppertal

Many interesting problems in the representation theory of finite groups aim to relate information about a group's structure to information that can be obtained from the character table. One of these problems is the recent question of Lyons–Martínez–Navarro–Tiep regarding the field of values of extensions of characters in principal blocks.

We will discuss how this question is related to understanding the distribution of characters of unipotent blocks of almost simple groups of Lie type.

Cyclotomic level maps and associated varieties for simple affine vertex algebras

Peng Shan
Tsinghua University

We will introduce cyclotomic level maps on the set of nilpotent orbits of a semi-simple Lie algebra and on the conjugacy classes of its Weyl group, and explain their compatibilities with Lusztig's map between these two sets. Then we explain a conjecture about associated varieties of simple affine vertex algebras at nonadmissible levels using these cyclotomic level maps, and some evidences of it. Time permitting, we will also discuss relationship with affine Springer theory. This is based on joint work with Wenbin Yan and Qixian Zhao.

McKay's equality on p'-characters, the landscape after the proof, II

Britta Späth
Bergische Universität Wuppertal

In this second talk we focus mainly on the aspects more specific to McKay's statement and leading to the ultimate proof by the speaker and Marc Cabanes. The crucial step is here the local part of the three-part criterion described in the first talk. This is where one must prove for the normalizer of a Sylow ℓ-subgroup some statements similar to the ones asked from the quasisimple group G itself. The choice of an overgroup of those normalizers is a crucial one. We explain how a certain integer d and associated Broué-Malle so-called Sylow d-tori, already routinely used to build the Sylow subgroups, are of a crucial importance along with their normalizer N, leading to the conditions called A(d) and B(d) in the course of the proof. In types other than D the checking involves the global condition A(∞) for groups of smaller ranks along some character theory of extended Weyl groups. This looks less practicable in type D so we have used a maximal subgroup M containing N and a more complete descrition of Irr(G) beyond the sole condition A(∞). This idea is also used in other checkings, e.g. the ongoing checking of the Alperin-McKay conjecture through the CFSG.

Modular reduction of nilpotent orbits

Jay Taylor
University of Manchester

We consider a split connected reductive algebraic $\mathbb{Z}$-group G and V a G-module which is either the Lie algebra $\mathfrak{g}$ or its dual $\mathfrak{g}^*$. If $\Bbbk$ is an algebraically closed field then, by base change, we get a group $G_\Bbbk$ and a corresponding module $V_\Bbbk$. Hesselink has defined a partition of the nullcone $\mathcal{N}(V_\Bbbk)$ of $V_\Bbbk$ into strata $\mathcal{N}(V_\Bbbk \mid \mathcal{O})$ which can be indexed, thanks to Clarke-Premet, by $G(\mathbb{C})$-orbits $\mathcal{O} \subseteq \mathcal{N}(\mathfrak{g}_\mathbb{C})$, such that $\mathcal{N}(\mathfrak{g}_\mathbb{C} \mid \mathcal{O}) = \mathcal{O}$. Each stratum is a union of $G(\Bbbk)$-orbits.

In this talk I will describe joint work with Adam Thomas (Warwick) which produces for each orbit $\mathcal{O} \subseteq \mathcal{N}(\mathfrak{g}_\mathbb{C})$, via a case-by-case analysis, integral representatives $e \in V \cap \mathcal{N}(V_\mathbb{C} \mid \mathcal{O})$ whose reduction $e_\Bbbk \in \mathcal{N}(V_\Bbbk \mid \mathcal{O})$ is well-behaved for every algebraically closed field $\Bbbk$. There are three possibilities for what well-behaved can mean and we treat all three.

Character estimates for finite simple groups and applications

Pham Huu Tiep
Rutgers University

Given the current knowledge of complex representations of finite (quasi)simple groups, obtaining good estimates for their character values still remains a difficult problem, a satisfactory solution of which would have significant implications in a number of applications, in and outside of group theory. We will report on recent results, obtained by the speaker and his collaborators, that produce such character bounds, and discuss some such applications.

On growth and complexity functions

Efim Zelmanov
Southern University of Science and Technology

I will discuss history and recent results concerning (i) growth functions of groups, algebras, monoids and languages, (ii) complexity functions of infinite sequences.

Green correspondence and weight conjecture

Jiping Zhang
Peking University

Let G be a finite group and k, a large enough field of characteristic $p > 0$ with $p \mid |G|$. We prove that the Green correspondence establishes a bijection between irreducible modules and weights for a finite group G over the field k if and only if the source of every irreducible kG-module is the trivial module.

Abstracts of contributed talks

Relative M-groups and relative M_p-groups

Xiaoyou Chen
Henan University of Technology

Let G be a finite group, p be a prime and let IBr(G) be the set of irreducible (p-)Brauer characters of G. If every Brauer character $\varphi \in$ IBr(G) is monomial, then G is said to be an M_p-group. We introduce some results about M_p-groups and relative M_p-groups in this talk.

The blockwise Galois Alperin weight conjecture for symmetric and alternating groups and their double covering groups

Yucong Du
Chongqing University

Recently, it is of great interest to characterize the fields of values of the irreducible ordinary and Brauer characters of a finite group. The blockwise Galois Alperin weight conjecture, proposed by Navarro in 2004, implies that the fields of values of irreducible Brauer characters in a block can be locally determined. In this talk, we prove that the blockwise Galois Alperin weight conjecture holds for symmetric groups, alternating groups and their double covering groups, which provides substantial evidence for this conjecture. This is joint work with Xin Huang and Jiping Zhang.

A reduction of the Galois Alperin weight conjecture and its blockwise version

Qulei Fu
Southern University of Science and Technology

Inspired by the work of Navarro and Turull, we formulate a strengthened version of the Alperin weight (AW) conjecture, which we call the Galois Alperin weight (GAW) conjecture. This conjecture asserts that the AW bijection can be chosen to be simultaneously equivariant under the actions of group automorphisms and Galois automorphisms. We then reduce the GAW conjecture—extending the reduction of the original AW conjecture due to Navarro and Tiep—to an inductive condition for (quasi-)simple groups, and we verify this condition in several cases. Moreover, we refine the conjecture to a blockwise version and obtain a corresponding reduction.

Finite simple groups as automorphism groups of non-associative algebras and colored graphs

Ilya Gorshkov
Sobolev Institute of Mathematics

Axial algebras are a class of non-associative commutative algebras whose properties are defined by a fusion law. When this fusion law is 2-graded, the algebra has a naturally associated automorphism group generated by involutions, and thus axial algebras are related to the theory of finite simple groups. Examples of axial algebras include finite-dimensional simple Jordan algebras and the Griess algebra. In this talk, we will generalize the theory of axial algebras and construct colored graphs whose automorphisms are finite almost simple groups.

Hyperfocal subalgebras of blocks with Frobenius hyperfocal inertial quotients

Xueqin Hu
Central China Normal University

In this talk, we first introduce the definition of a block with a Frobenius hyperfocal inertial quotient which is an analogue of a block with a Frobenius inertial quotient at the hyperfocal level. Blocks with cyclic hyperfocal subgroups and blocks with Klein four hyperfocal subgroups are of this type. We will generally investigate the structure of the hyperfocal subalgebra of the block of this type in terms of the stable category. As applications, the structures of the derived categories and characters of hyperfocal subalgebras with cyclic hyperfocal subgroups or Klein four hyperfocal subgroups are obtained. Consequently, we can verify Broué's abelian defect group conjecture for blocks with Klein four hyperfocal subgroups and the forward direction of 'hyperfocal height zero' version of Brauer's height zero conjecture proposed by Kessar, Linckelmann and Navarro for blocks with cyclic hyperfocal subgroups or Klein four hyperfocal subgroups. This is a joint work with Kun Zhang and Yuanyang Zhou.

A new generalization of the McKay conjecture for p-solvable groups

Ping Jin
Shanxi University

In this talk we will present a new generalization of the McKay conjecture for p-solvable groups by using the theory of self-stabilizing pairs founded by M. L. Lewis, as well as some new results on π-special characters due to I. M. Isaacs and G. Navarro. This is joint work with Huimin Chang.

Maximal solvable subgroups

Mikko Korhonen
University of Tampere

A subgroup of a group G is said to be maximal solvable if it is maximal among the solvable subgroups of G. In his 1870 Traité, Jordan gave a classification of the maximal solvable subgroups of symmetric groups. The classification reduces to the primitive case, which is equivalent to the problem of classifying maximal irreducible solvable subgroups of $\mathrm{GL}(d, p)$, where p is a prime. In $\mathrm{GL}(d, p)$, the problem is reduced to the case of primitive irreducible solvable subgroups. These subgroups are then constructed in terms of maximal irreducible solvable subgroups of general symplectic groups $\mathrm{GSp}(2k, r)$ (r prime) and orthogonal groups $\mathrm{O}^{\pm}(2k, 2)$.

In this talk, we will discuss Jordan's classification in modern terms. More generally, we consider the complete classification of maximal irreducible solvable subgroups of classical groups such as $\mathrm{GL}(n, q)$, $\mathrm{GSp}(n, q)$, and $\mathrm{GO}(n, q)$, where q is a power of a prime. From the classification we also get a recursive construction of the maximal irreducible solvable subgroups, and this works efficiently when implemented on a CAS such as Magma or GAP. We will also discuss the analogous problem for linear algebraic groups over algebraically closed fields.

On fusion systems of blocks of finite groups and stable equivalences of Morita type

Conghui Li
Southwest Jiaotong University

Brauer pairs, introduced by J. Alperin and M. Broué, are essential local data of blocks. Fusion systems is a method to organize these local data of blocks. Nilpotent blocks, defined by M. Broué and L. Puig, can be characterized by fusion systems. Stable equivalences of Morita type, defined also by M. Broué, is a useful tool in the study of blocks of finite groups. L. Puig proved that stable equivalences of Morita type with endopermutation sources between blocks induce isomorphisms of fusion systems. In this talk, we share an observation that stable equivalences of Morita type between blocks (not necessarily with endopermutation sources) induce isomorphisms of some quotient fusion systems.

Categorical action on finite classical groups and its applications: positive characteristic

Pengcheng Li
The University of Hong Kong

Categorification has played a prominent role in the recent advances in representation theory. The pioneering work of Chuang and Rouquier introduced the notion of an $\mathfrak{sl}_2$-categorification and, using this new tool, solved Broué's abelian defect group conjecture for the symmetric groups S_n and the finite general linear groups $\mathrm{GL}_n(q)$. For the finite classical groups $\mathrm{GU}_n(q)$, $\mathrm{SO}_{2n+1}(q)$ and $\mathrm{Sp}_{2n}(q)$, Dudas–Varagnolo–Vasserot similarly constructed an $\mathfrak{sl}_I$-categorification on unipotent blocks and verified Broué's conjecture for these blocks at linear primes.

In ongoing work, we construct a categorical double quantum Heisenberg action on the representation category of finite classical groups $\mathrm{O}_{2n+1}(q)$, $\mathrm{Sp}_{2n}(q)$ and $\mathrm{O}^{\pm}_{2n}(q)$, and explore its applications in positive characteristic. By employing the modulo ℓ reduction of the colored weight functions $\mathbb{O}^{+}(u)(\cdot)$ and $\mathbb{O}^{-}(v)(\cdot)$, we complete the determination of the ℓ-block distribution for the characters of $\mathrm{Sp}_{2n}(q)$ and $\mathrm{O}^{\pm}_{2n}(q)$, and prove that Broué's abelian defect group conjecture holds for all blocks of $\mathrm{Sp}_{2n}(q)$ and $\mathrm{O}^{\pm}_{2n}(q)$ at linear primes. This is a joint work with Peng Shan and Jiping Zhang.

Character degrees and group structure

Yanjun Liu
Jiangxi Normal University

The study of character degrees is an interesting topic and has a long history in the representation theory of finite groups. In this talk, we shall focus on a new situation on character degrees and give a classification of the involved nonsolvable groups.

Galois action and block

J. Miquel Martínez
Universitat de València

Many of the local-global conjectures in the representation theory of finite groups admit refinements when considering the fields of values of characters in blocks (frequently studied through the action of certain Galois automorphisms). In this talk, we will review older and newer problems as well as recent results on the action of Galois automorphisms on characters in blocks and their influence on local structure.

Recent progress on the modular isomorphism problem

Taro Sakurai
Chiba University

The modular isomorphism problem was a long-standing open problem on the group algebra FG of a finite p-group G over a field F of positive characteristic p. It asks whether $FG \cong FH$ implies $G \cong H$ or not.

The first counterexamples to the modular isomorphism problem are recently discovered by García-Lucas, Margolis and del Río. Accordingly, the smallest order of the counterexamples turns out to be $2^9 = 512$. Nevertheless, the only counterexamples that are discovered so far are for $p = 2$ and the odd prime cases remain open.

To understand where the counterexamples come from, we study the class of two-generated finite 2-groups with dihedral central quotient. As a result, we discover new counterexamples and a simple proof.

The new counterexamples per se are also discovered independently by Bagiński and Zabielski. However, our theorem shows that, within the above class of groups, there are the only counterexamples.

We also have positive results in various forms. This talk is based on join works with D. García-Lucas, L. Margolis, Á. del Río and M. Stanojkovski.

Cocenter of Hecke algebras of crystallographic Coxeter groups

Felix Schremmer
The University of Hong Kong

Hecke algebras are ubiquitous in the representation theory of finite and p-adic Lie groups. We study the cocenter of the Hecke algebra of a crystallographic Coxeter group using representation theoretic tools as well as orbital integrals in Kac-Moody groups. This is joint work in progress with Xuhua He.

About the inductive Feit condition

Carlos Tapp
Rutgers University

In 1980, Walter Feit conjectured that for any finite group G, if one considers the conductor of any fixed irreducible character of G, then there exists an element in G of order precisely that conductor. Recently, this was reduced to a question about simple groups by R. Boltje, A. Kleshchev, G. Navarro, and P. Tiep. In this talk, I will present recent progress on the inductive Feit condition, with an emphasis on low-rank groups of Lie type.

Minimal polynomials in spin representations of symmetric and alternating groups

Velmurugan S
Indian Institute of Science, Bangalore

Let G be a double cover of the symmetric or alternating group. Let $g \in G$ and let ρ be any faithful irreducible representation of G. In this talk, we discuss the description of the minimal polynomial of $\rho(g)$. This is a joint work with Amritanshu Prasad and Alexey Staroletov.

On the solvability of the Lie algebra $\mathrm{HH}^1(B)$ for blocks of finite groups

Jialin Wang
City St George's University of London

We give some criteria for the Lie algebra of first degree Hochschild cohomology of the twisted group algebra, i.e. $\mathrm{HH}^1(k_\alpha(P \rtimes E))$, to be solvable, where P is a finite abelian p-group, E is an abelian p'-subgroup of $\mathrm{Aut}(P)$ and $\alpha \in Z^2(E; k^\times)$ inflated to $P \rtimes E$ via the canonical surjection $P \rtimes E \to E$. As a special case, this gives the criterion to the solvability of the Lie algebra $\mathrm{HH}^1(B)$ where B is a p-block of a finite group algebra with abelian defect P and inertial quotient E.

The character codegree ratio of finite groups

Quanfu Yan
Peking University

Motivated by the definition of the character degree ratio introduced by I.M. Isaacs, we define in this paper a dual concept, the character codegree ratio $\mathrm{rat}_c(G)$ of a finite group G, which is the largest ratio of codegrees of two nonprincipal irreducible characters of G. We first show that if G has no composition factor isomorphic to $\mathrm{PSL}_2(q)$, then $|G : \mathrm{Rad}(G)| \leq \mathrm{rat}_c(G)^{10}$, where $\mathrm{Rad}(G)$ denotes the solvable radical of G. We also classify finite groups with $\mathrm{rat}_c(G) < 2$. As a consequence of this classification, we can determine the structure of finite groups whose nontrivial character codegrees are consecutive integers. Furthermore, we show that the derived length and the Fitting height of a finite solvable group is bounded by a doubly logarithmic function in terms of $\mathrm{rat}_c(G)$.

Examples of inertial blocks

Kun Zhang
Hubei University

A block is called inertial if it is basic Morita equivalent to a block with normal defect group. Let p be a prime. We proved that any p-block with abelian defect groups is inertial if it has only one irreducible Brauer character orbit. Let D be a 2-group of the form $C_{2^{n_1}} \times C_{2^{n_2}} \times \cdots \times C_{2^{n_t}}$ where $t \geq 1$ and $n_i \geq 2$ for all i. We also proved that all 2-blocks with defect group D are inertial.